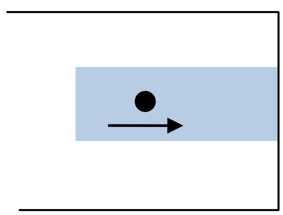
Teacher notes
Topic B

Another derivation of $P = \frac{1}{3}\rho c^2$.

As usual, we imagine a molecule of mass *m* hitting the right wall of a container with speed v_x and bouncing back with the same speed, an elastic collision.



The change in momentum is $2mv_x$. If the collision lasts for a time τ, the average force exerted by the molecule is $\frac{2mv_x}{\tau}$.

Now the container has *n* molecules per unit volume. Imagine a cylinder of length $v_x T$. In time *T* all the molecules within the cylinder will hit the wall. So the total force exerted is

$$\frac{2mv_x}{\tau} \times nAv_x T$$

where *A* is the cross-sectional area of the cylinder. However, not every molecule in this cylinder will hit the wall at the same time. Each molecule in the cylinder is in contact with the wall for a time τ so the actual force is only a fraction $\frac{\tau}{T}$ of the expression above. Hence

$$\frac{2mv_x}{\tau} \times nAv_x T \times \frac{\tau}{T} = 2mnAv_x^2$$

We expect half the molecules in the cylinder to be going to the right and the other half to the left so that the pressure is $P = \frac{1}{2} \times 2mnv_x^2 = mnv_x^2$. But $mn = \rho$, the density of the gas. Different molecules have different speeds so we need to take an average of the squares of the speeds to get the average pressure: $\overline{P} = mn\overline{v}_x^2$. Further, $\overline{v}_x^2 = \overline{v}_y^2 = \overline{v}_z^2$ since all directions are equivalent and $\overline{v}_x^2 + \overline{v}_y^2 + \overline{v}_z^2 = c^2$ so that $\overline{v}_x^2 = \frac{1}{3}c^2$. Hence

$$\overline{P} = \frac{1}{3}\rho c^2 .$$